and

$$\overline{A_i} = \frac{\beta + (1-\beta)\,\overline{K_D}(t'_p - t'_g)}{1 - (1-\beta)t'_g\ t'_R} \qquad (55')$$

where, for the case of the CSTR reactor and regenerator of equal size,

$$T'_G(t) = T'_R(t) = \lambda e^{-\lambda t}$$

$$t'_R = \frac{\lambda}{k_r + p + \lambda}$$

$$t'_p = \frac{\lambda}{p + \lambda}$$

$$t'_g = \frac{\lambda}{k_g + p + \lambda}$$

and

$$\overline{K_D} = \frac{\beta\lambda(p + \lambda)}{p^2 + 2p\lambda + \beta\lambda}$$

The approach of Equations (40) and (42) is not exact, but alternate deactivation and reactivation greatly reduces interaction in the second half of a cycle. Thus, a comparison of mean activities calculated from Equations (50′) and (55′) with those from Petersen's equations (50) and (55) showed the latter to be low by about 25% for the once-through makeup catalyst and by about 3% for the total stream (where $p = k_r = k_g = \lambda$, $\beta = 0.1$).

It is worth noting that although the authors, like Petersen, chose to express activity as a function of time, or age, the problem might have been formulated as proposed by Rudd (*Can. J. Chem. Eng.*, **40**, 197, [Oct. 1962]).

Correlation of Liquid Slug Velocity and Frequency in Horizontal Cocurrent Gas-Liquid Slug Flow

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Liquid slug velocities and frequencies have been measured for the system carbon dioxide-water in a ¾ in. diameter tube. The gas and liquid were introduced to the tube using a simple tee mixer, located approximately 300 pipe diameters upstream of the measuring point. This distance was chosen to ensure that fully developed slug flow pattern would be obtained. All runs were performed at essentially atmospheric pressure and 25°C. A detailed description of the complete flow apparatus is given elsewhere (*1*, *2*).

SLUG VELOCITY

The slug velocity measuring procedure has been used by Hubbard (*3*) and consisted of measuring the time lag between the pressure pulses for a given slug at two points in the test section. Pressures were measured using high response strain gauge transducers mounted into small wells located on the underside of the test section. The transducer voltage outputs were amplified and recorded on a two channel, high response, strip chart recorder.

The slug velocity values reported each represent the average of about ten determinations for a given set of gas and liquid flow rates. Individual values varied by about ± 18% from the mean. The no-slip velocity is defined as the sum of the gas and liquid superficial velocities. Hence,

$$V_{ns} = V^0{}_L + V^0{}_G \qquad (1)$$

Figure 1 shows the measured slug velocity plotted against the no-slip velocity. The line of best fit through the data is given by,

$$V_s = 1.35\ V_{ns} \qquad (2)$$

Hubbard (*3*) has obtained the relation

$$V_s = 1.25\ V_{ns} \qquad (3)$$

from his slug flow model. However, his data shows better agreement, particularly at the higher slug velocities with Equation (2) than with (3) as shown in Figure 2.

As a consequence of Equation (3), Hubbard's theory also predicts that the true average gas velocity is given by the relation,

$$V_G = 1.11\ V_{ns} \qquad (4)$$

Based on the Hughmark correlation (*4*) for slug holdup,

$$V_G = 1.22\ V_{ns} \qquad (5)$$

for $N_{ReLs} > 400{,}000$, and for $N_{ReLs} < 400{,}000$, the constant is somewhat larger, being approximately proportional to $(N_{ReLs})^{-1/4}$. [Note: N_{ReLs} = liquid slug Reynolds number $= d\ (V^0{}_L + V^0{}_G)\ \rho_L/\mu_L$]. In addition, Nicklin, et al. (*5*) claim that for horizontal slug flow

$$V_G = 1.20\ V_{ns} \qquad (6)$$

With the slug velocity given by Equation (2), Hubbard's theory predicts

$$V_G = 1.19\ V_{ns} \qquad (7)$$

which is in better agreement with other published data.

SLUG FREQUENCY

Slug frequencies were measured by two techniques. The first consisted of simply counting, from visual observation, the number of liquid slugs passing a given point in the test section, over a period of time measured with a stopwatch. The second method consisted of counting the number of slug pressure pulses recorded on the strip chart recorder of the pressure measuring system, for a given period of time. Good agreement was obtained between the two methods, and either one is considered satisfactory. The measured slug frequencies, and the corresponding slug velocities are each the average of about 10 individual observations,

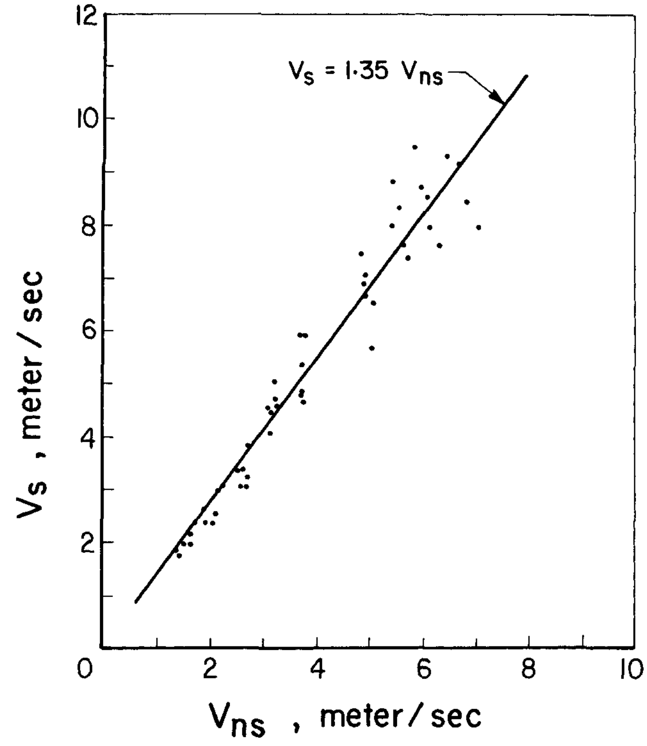

Fig. 1. Measured slug velocity vs. no-slip velocity.

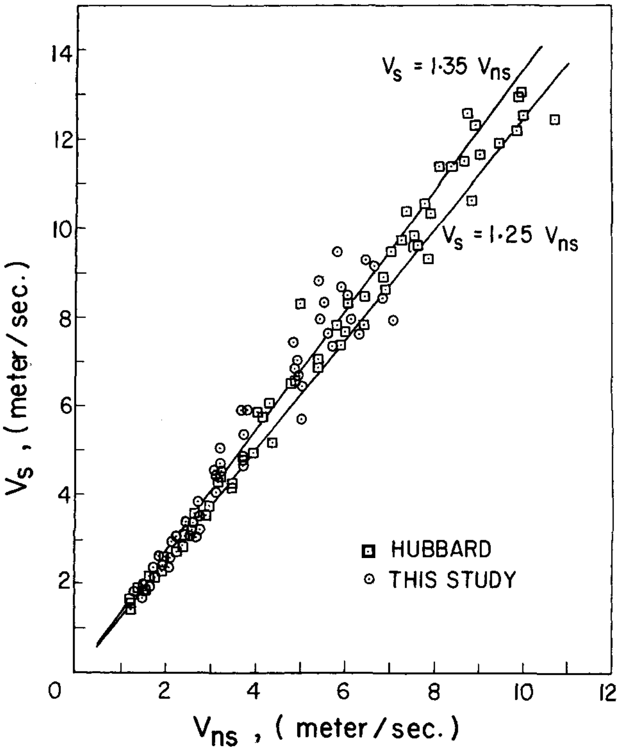

Fig. 2. Measured slug velocity vs. no-slip velocity.

which showed a maximum deviation of about 20% from the mean value. Figure 3 shows the slug frequency data plotted against the slug velocity. Figure 4 is a plot of the corresponding data obtained by Hubbard (3). Hubbard attempted to correlate his frequency data using a method proposed by Semenov (7), in which a reduced slug frequency is defined as

$$N_r = \frac{d}{(V_{ns} - V'_{ns})\nu_s} \tag{8}$$

where N_r = reduced slug frequency, d = tube diameter, ν_s = slug frequency, V_{ns} = no-slip or superficial mixture velocity and V'_{ns} is an experimentally determined minimum mixture velocity at which slug flow begins, and is a function of the inlet volumetric gas fraction. N_r is then plotted against the inlet gas volumetric fraction. However, Hubbard found that his data gave very poor agreement with this correlation, and concluded that Equation (8) did not represent a general correlating technique.

One of the immediate observations that can be made from the data shown in Figures 3 and 4, is that in both cases, over almost the total range of liquid flow rates, the curves exhibit a minimum at $V_s \approx 6$ m./sec. Also, the shape of the curves suggests a velocity dependence of the form,

$$\nu_s = \frac{A}{V_s} + B\,V_s + C \tag{9}$$

On differentiating Equation (9) with respect to V_s, and setting $d\,\nu_s/d\,V_s = 0$, we obtain,

$$\frac{d\,\nu_s}{d\,V_s} = -\frac{A}{(V_s^*)^2} + B = 0 \tag{10}$$

that is

$$A = (V_s^*)^2\,B$$

where V_s^* is the slug velocity at the minimum point in the ν_s vs. V_s curve.

Hence, substituting Equation (10) into (9), we obtain

$$\nu_s = B\left[\frac{(V_s^*)^2}{V_s} + V_s\right] + C \tag{11}$$

Further examination of Figures 3 and 4 indicates a direct dependence of ν_s on the superficial liquid velocity, V^0_L, and an inverse dependence of ν_s on the tube diameter, d. This suggests that the slug frequency should correlate with a form of Froude number, defined as,

$$(N_{Fr})_{\text{slug}} = \frac{V^0_L}{g\,d}\left[\frac{(V_s^*)^2}{V_s} + V_s\right] \tag{12}$$

Hubbard's data and the data from the present study are shown in Figure 5 plotted against $(N_{Fr})_{\text{slug}}$, with the value of (V_s^*) taken to be 6 m./sec The data collapse very well, with the best fit line given by

$$\nu_s = 0.0157\,(N_{Fr})_{\text{slug}}^{1.2} \quad \text{sec.}^{-1} \tag{13}$$

with a standard deviation of 15.8%. Hence the constant, C, can be taken as zero.

The data used to obtain Equation (13) contain a threefold variation in V^0_L, a twofold variation in d, and a sixfold variation in V_s. By combining Equations (2), (12), and (13) we obtain

$$\nu_s = 0.0226\left[\frac{V^0_L}{g\,d}\left(\frac{19.75}{V_{ns}} + V_{ns}\right)\right]^{1.2} \quad \text{sec.}^{-1} \tag{14}$$

and ν_s is expressed in terms of quantities which are all readily available *a priori.* [Quantities in Equation (14) are in meters and seconds.]

Equation (14) must, at this stage, be considered a preliminary result as it is based on only two sets of data. It is possible that further data, for other gas-liquid systems and tube diameters, may exhibit slug frequency minima at values other than 6 m./sec., and hence some means of predicting the value of the minimum from the fluid properties, etc., will then be required. However, Equation (14) appears to be the only existing correlation for slug frequencies, and should be satisfactory for prediction purposes within the ranges of the variables in Hubbard's and this study.

Future work should be directed toward (a) the verification of the use of the slug Froude number as a general slug

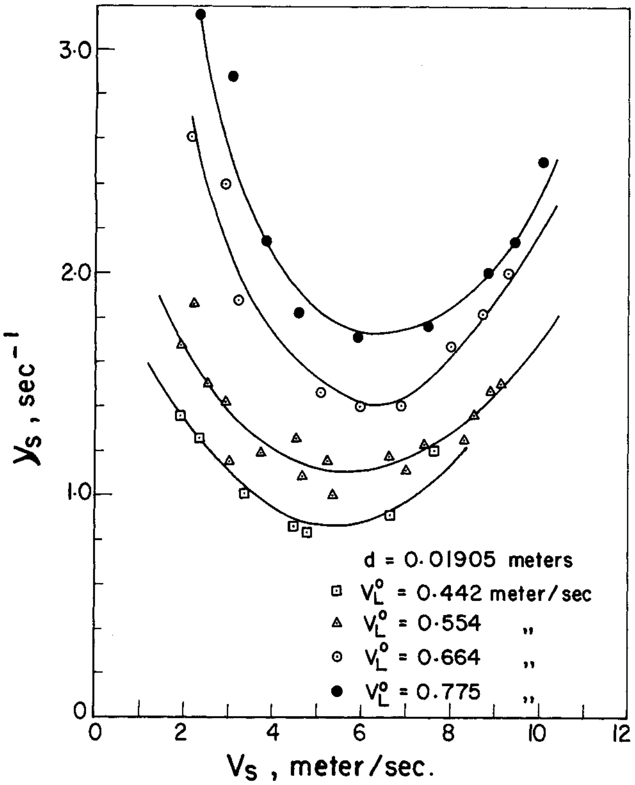

Fig. 3. Slug frequency vs. measured slug velocity (this study).

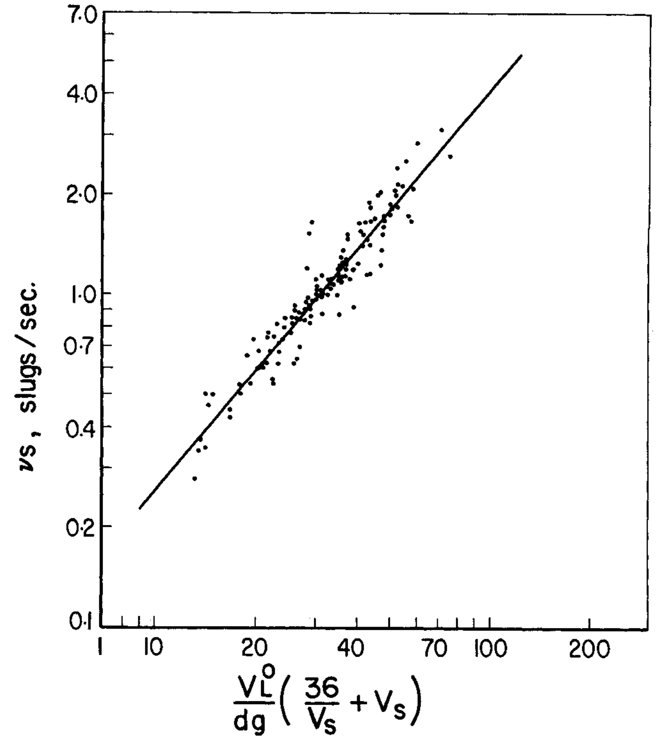

Fig. 5. Slug frequency vs. slug Froude number for $(V_s^*)^2 = 36$ m./sec.

frequency correlating technique; and (b) obtaining a satisfactory slug density correlation or theory. The ability to predict these two quantities will permit the use of Hubbard's slug flow model (*3, 7*) to calculate pressure drops, and other characteristic parameters in this flow regime. The model can probably also be extended to include mechanistic predictions for both heat and mass transfer in horizontal slug flow.

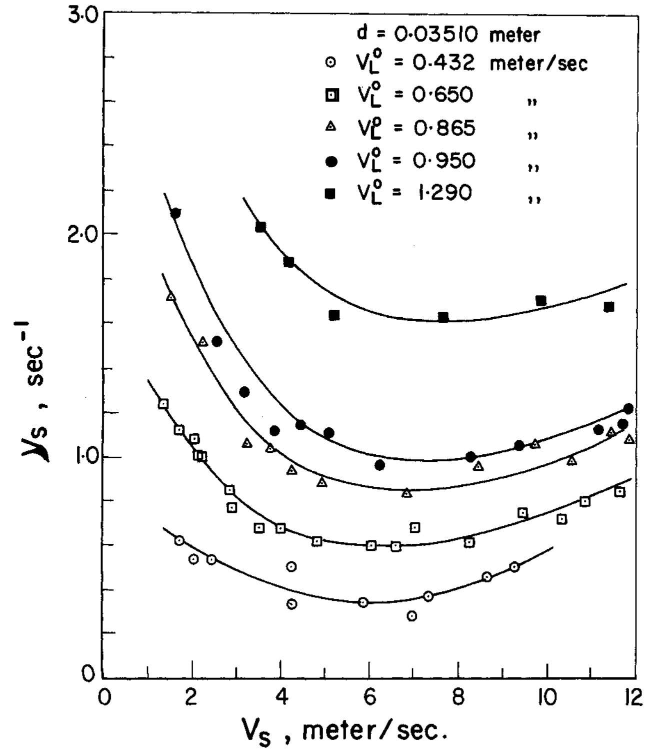

Fig. 4. Slug frequency vs. measured slug velocity (Hubbard).

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NOTATION

$A, B, C,$ = constants in Equation (9)
d = test section I.D., meter
g = gravitational constant, m./sec.^{2}
N_r = reduced slug frequency, defined by Equation (8)
V^0_L, V^0_G = superficial liquid and gas velocities respectively, m./sec.
V_G = true average gas velocity, m./sec.
V_{ns} = no-slip velocity in test section, defined by Equation (1)
V_s = slug velocity, m./sec.
V_s^* = slug velocity at minimum slug frequency, m./sec.
$(N_{Fr})_{slug}$ = slug Froude number, defined by Equation (12)
ρ_L = liquid density, g./cu.m.
μ_L = liquid viscosity, g./m./sec.
ν_s = liquid slug frequency, sec.$^{-1}$

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